Product and Quotient Rules

$(x+4)(x-3)$

The Product Rule

The derivative of f(x) times g(x)....

The first function times $\frac{d}{dx} g(x)$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

This can be applied to more than two functions.

$$\frac{d}{dx}[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

Find the derivative of f(x)g(x)

$$f(x) = 3x - 2x^2$$

$$g(x) = 5 + 4x$$

$f(x)g(x) = (3x - 2x^2)(5 + 4x)$

$\underbrace{(3x - 2x^2)}_{}^{f(x)} \overset{g'(x)}{(4)} + \overset{f'(x)}{(3 - 4x)} \overset{g(x)}{(5 + 4x)}$

$12x - 8x^2 + (15 - 8x - 16x^2)$

$12x - 8x^2 + 15 - 8x - 16x^2$

$\boxed{-24x^2 + 4x + 15} = \frac{d}{dx}(f(x)g(x))$

Find the derivative of:

$$\overset{f(x)}{3x^2}\;\overset{g(x)}{\sin x}$$

$y = 3x^2 \sin x$

$(3x^2)(\cos x) + (6x)(\sin x)$

$y' = 3x^2 \cos x + 6x \sin x$

$$y = \overset{f(x)}{2x}\overset{g(x)}{\cos x} - \underbrace{2\sin x}$$

$$y' = \overset{f(x)}{(2x)}\overset{g'(x)}{(-\sin x)} + \overset{f'(x)}{(2)}\overset{g(x)}{(\cos x)} - 2\cos x$$

$$-2x\sin x + \cancel{2\cos x} \cancel{-2\cos x}$$

$$\boxed{y' = -2x\sin x}$$

$y = x^2 \sin x \cos x$

$y' = 2x \sin x \cos x + x^2 \cos x \cos x + x^2 \sin x(-\sin x)$

$y' = 2x \sin x \cos x + x^2 \cos^2 x - x^2 \sin^2 x$

If:

$x^2 \sin^2 x + x^2 \cos^2 x$

then

$x^2(\sin^2 x + \cos^2 x)$

x^2

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \qquad g(x)\neq 0$$

$$y = \frac{5x-2}{x^2+1}$$

$$\frac{(5 \cdot x^2+1) - (2x \cdot 5x-2)}{(x^2+1)^2}$$

$f'(x) = 5$

$g'(x) = 2x$

$$\frac{5x^2+5-10x^2+4x}{x^4+2x^2+1}$$

$$\frac{-5x^2+4x+5}{x^4+2x^2+1}$$

Find the equation of a tangent line to the graph of:

$$y = \frac{3 - \frac{1}{x}}{x+5} \quad at\ (-1,1)$$

$y = \frac{3 - \frac{1}{x}}{x+5} \cdot \frac{x}{x} = \frac{3x-1}{x^2+5x}$

① find $\frac{d}{dx}$

$y' = \frac{-3x^2+2x+5}{(x^2+5x)^2}$

② find slope

Slope

$y'(-1) = \frac{-3(-1)^2+2(-1)+5}{((-1)^2+5(-1))^2}$

$m = 0$

③ point-slope form

$y-y = m(x-x)$

$y-1 = 0(x+1)$

$y-1 = 0$

$\boxed{y=1}$ eq. of tangent line

$$y = \frac{-3(3x - 2x^2)}{7x}$$

$$y = \frac{-3}{7} \cdot \frac{(3x-2x^2)}{x}$$

$$y = \frac{-3}{7} \cdot \frac{\cancel{x}(3-2x)}{\cancel{x}}$$

$$y = \frac{-3}{7}(3-2x)$$

$$y' = \frac{-3}{7}(-2)$$

$$y' = \frac{6}{7}$$

We know...

$$\frac{d}{dx}[sinx] = cosx \qquad \frac{d}{dx}[\cos x] = -\sin x$$

From this, we can use the quotient rule to differentiate other trig functions.

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\underset{\frac{1}{\cos x}}{\sec x}] = \underbrace{\sec x \tan x} \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$y = x - \tan(x)$

$y' = 1 - \sec^2 x$

$y = x\sec(x)$

$x(\sec x \tan x) + 1(\sec x)$

$\sec x + x\sec x\tan x$

$\sec x(1 + x\tan x)$

HW

1-19 odd, 23, 25, 29, 35,

39, 47-53 odd

$$y = \frac{1 - \cos x}{\sin x}$$